

SUPERFIELD COVARIANT QUANTIZATION WITH BRST SYMMETRY

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We generalize the method of superfield Lagrangian BRST quantization in the part of the gauge-fixing procedure and obtain a quantization method that can be considered as an alternative to the Batalin–Vilkovisky formalism.

1 Introduction

In previous years it has been realized that two principles are essential in the formulation of appropriate quantum versions of a classical theory considered in the covariant, i.e. Lagrangian, approach, namely, the invariance under the BRST transformations [1], and the gauge-independence of the S -matrix.

It is well-known that the first principle can be replaced by the requirement of BRST–antiBRST symmetry [2], which also leads to possible versions of quantization methods, such as the $Sp(2)$ -covariant quantization [3], the triplectic quantization [4], or the extended superfield BRST quantization [5].

For the first time, the two above-mentioned principles were realized in the framework of the BV quantization method [6] for general gauge theories, proposed by Batalin and Vilkovisky.

It should be noted that the requirements of (extended) BRST symmetry and gauge-independence are two independent principles, which can be exemplified by the method of $OSP(1, 2)$ -covariant quantization [7] that satisfies the requirement of extended BRST symmetry, while violating the requirement of gauge-independence.

Note also that there is arbitrariness in the realization of the above principles, which implies that there can be off-shell different quantum versions of a classical theory. The existence of this arbitrariness may be exemplified by different extensions of the BV quantization method (see, e.g., [8]).

The purpose of this report is to demonstrate the fact that extending the method of the BV quantization in the part of gauge-fixing, while retaining the principles of BRST symmetry and gauge-independence, can lead to off-shell different quantum theories. To do so, we consider the superfield form [9] of the BV quantization rules, which provides another realization of the principles

of BRST symmetry and gauge-independence. By extending [9] in the part of gauge-fixing, we obtain a quantization scheme [10] that differs from a natural generalization of the gauge-fixing procedure inherent in the BV quantization.

2 Batalin–Vilkovisky Formalism

Let us remind the basic ingredients of the BV quantization method for general gauge theories [6].

The starting point is a classical gauge-invariant action $S_0(A)$ of fields A^i . The action may belong to the class of irreducible or reducible theories, with a closed or open algebra of gauge transformations. The complete configuration space ϕ^A is introduced, spanned by the initial fields A^i , the ghost fields, and the auxiliary fields. The content of the complete configuration space is determined by the properties of the initial theory, i.e. by the fact whether the theory is an irreducible or reducible one. Note that the explicit structure of the complete configuration space is not essential for the following considerations.

With each field ϕ^A , a corresponding antifield ϕ_A^* is associated, having the opposite Grassmann parity, $\varepsilon(\phi_A^*) = \varepsilon_A + 1$. In the space of the field-antifield variables one defines the antisymplectic operation $(\ , \)$ called the antibracket

$$(F, G) = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi_A^*} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} \frac{\delta G}{\delta \phi^A} \frac{\delta F}{\delta \phi_A^*} \quad (1)$$

and the nilpotent operator Δ

$$\Delta = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_A^*}, \quad \Delta^2 = 0. \quad (2)$$

The basic object of the BV quantization is the quantum action $S = S(\phi, \phi^*)$ that satisfies the generating equation

$$\frac{1}{2}(S, S) = i\hbar \Delta S, \quad (3)$$

or equivalently

$$\Delta \exp \left\{ \frac{i}{\hbar} S \right\} = 0, \quad (4)$$

with the boundary condition

$$S|_{\phi^*=0} = S_0.$$

The vacuum functional Z is defined as an integral over the fields of the complete configuration space ϕ^A ,

$$Z = \int d\phi \exp \left\{ \frac{i}{\hbar} S_{\text{eff}}(\phi) \right\}, \quad (5)$$

where

$$S_{\text{eff}}(\phi) = S_{\text{ext}}(\phi, \phi^*)|_{\phi^*=0}. \quad (6)$$

In eq. (6), $S_{\text{ext}} = S_{\text{ext}}(\phi, \phi^*)$ is the gauge-fixed quantum action constructed as the symmetry transformation

$$\exp\left\{\frac{i}{\hbar}S_{\text{ext}}\right\} = \exp\left(-[\Delta, \Psi]_+\right) \exp\left\{\frac{i}{\hbar}S\right\}, \quad (7)$$

retaining the form of the generating equation (4)

$$\Delta \exp\left\{\frac{i}{\hbar}S_{\text{ext}}\right\} = 0, \quad (8)$$

and introduced with the help of the gauge fermion Ψ that removes the degeneracy of the functional integral (5).

Equivalently, the vacuum functional (5) can be represented in the form of an integral over the variables ϕ^A , ϕ_A^* , λ_A ,

$$Z = \int d\phi d\phi^* d\lambda \exp\left\{\frac{i}{\hbar}\left[S_{\text{ext}}(\phi, \phi^*) + \phi_A^* \lambda^A\right]\right\}, \quad (9)$$

where λ_A are additional fields with the Grassmann parity opposite to that of the fields ϕ^A , $\varepsilon(\lambda_A) = \varepsilon_A + 1$.

By virtue of eq. (8), the integrand in eq. (9) is invariant under the global supersymmetry transformations

$$\delta\phi^A = \lambda^A \mu, \quad \delta\phi_A^* = \mu \frac{\delta S_{\text{ext}}}{\delta\phi^A}, \quad \delta\lambda^A = 0 \quad (10)$$

with an anticommuting parameter μ . Eqs. (10) realize the transformations of BRST symmetry in the space of the variables ϕ^A , ϕ_A^* , λ_A .

It follows from eqs. (4), (7) and (9) that the vacuum functional $Z \equiv Z_\Psi$ does not depend on the choice of the gauge fermion, $Z_{\Psi+\delta\Psi} = Z_\Psi$, and hence, by the equivalence theorem [11], the corresponding S -matrix is gauge-independent.

When formulating the rules of the BV quantization, the choice of Ψ in the symmetry transformation (7) was made in the form of a functional depending on the fields, $\Psi = \Psi(\phi)$. In this case the corresponding vacuum functional (5) reduces to the well-known expression

$$Z = \int d\phi d\phi^* d\lambda \exp\left\{\frac{i}{\hbar}\left[S(\phi, \phi^*) + \left(\phi_A^* - \frac{\delta\Psi}{\delta\phi^A}\right)\lambda^A\right]\right\}. \quad (11)$$

3 Modified Superfield BRST Quantization

Recently, a closed superfield form of the BV quantization rules was proposed [9]. In this formalism all variables of the BV approach are combined into so-called superfields and super-antifields in a superspace spanned by space-time coordinates and a scalar Grassmann coordinate. The vacuum functional (11) of the BV formalism is contained in the superfield approach as a particular case of gauge-fixing and solutions of the generating equation.

We now generalize [10] the superfield BRST formalism [9] in such a way that the gauge is introduced by means of a bosonic functional depending on all variables of the superfield formalism, including the super-antifields. The gauge-fixing functional is subject to a generating equation similar to the equation that determines the quantum action.

Let us introduce a superspace (x^μ, θ) spanned by space-time coordinates x^μ , $\mu = (0, 1, \dots, D-1)$, and a scalar anticommuting coordinate θ . Let $\Phi^A(\theta)$ be a set of superfields $\Phi^A(\theta)$, accompanied by a set of the corresponding super-antifields $\Phi_A^*(\theta)$ with the Grassmann parities $\varepsilon(\Phi^A) \equiv \varepsilon_A$, $\varepsilon(\Phi_A^*) = \varepsilon_A + 1$. The superfields are subject to the boundary condition

$$\Phi^A(\theta)|_{\theta=0} = \phi^A. \quad (12)$$

Define the vacuum functional Z as the following integral:

$$Z = \int d\Phi d\Phi^* \rho[\Phi^*] \exp \left\{ \frac{i}{\hbar} \left(S[\Phi, \Phi^*] + X[\Phi, \Phi^*] + \Phi^* \Phi \right) \right\}. \quad (13)$$

In eq. (13), $S = S[\Phi, \Phi^*]$ is the quantum action determined by the generating equation

$$\frac{1}{2}(S, S) + VS = i\hbar \Delta S, \quad (14)$$

while $X = X[\Phi, \Phi^*]$ is the gauge-fixing functional that satisfies the equation

$$\frac{1}{2}(X, X) - UX = i\hbar \Delta X. \quad (15)$$

In eqs. (13), (14) and (15), we have used the antibracket $(\ , \)$ expressed in terms of arbitrary functionals $F = F[\Phi, \Phi^*]$, $G = G[\Phi, \Phi^*]$ by the rule

$$\begin{aligned} (F, G) = & \int d\theta \left\{ \frac{\delta F}{\delta \Phi^A(\theta)} \frac{\partial}{\partial \theta} \frac{\delta G}{\delta \Phi_A^*(\theta)} (-1)^{\varepsilon_A+1} - \right. \\ & \left. - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} (F \leftrightarrow G) \right\}. \end{aligned} \quad (16)$$

We have also used the operators Δ , V and U

$$\Delta = - \int d\theta (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \Phi^A(\theta)} \frac{\partial}{\partial \theta} \frac{\delta}{\delta \Phi_A^*(\theta)}, \quad (17)$$

$$V = - \int d\theta \frac{\partial \Phi_A^*(\theta)}{\partial \theta} \frac{\delta}{\delta \Phi_A^*(\theta)}, \quad (18)$$

$$U = - \int d\theta \frac{\partial \Phi^A(\theta)}{\partial \theta} \frac{\delta_l}{\delta \Phi^A(\theta)} \quad (19)$$

(derivatives with respect to θ are understood as acting from the left) as well as the functionals $\rho[\Phi^*]$ and $\Phi^*\Phi$

$$\rho[\Phi^*] = \delta \left(\int d\theta \Phi^*(\theta) \right), \quad (20)$$

$$\Phi^*\Phi = \int d\theta \Phi_A^*(\theta) \Phi^A(\theta). \quad (21)$$

The algebra of the above operators (17), (18) and (19) reads as follows:

$$\Delta^2 = 0, \quad U^2 = 0, \quad V^2 = 0, \quad (22)$$

$$\Delta U + U \Delta = 0, \quad \Delta V + V \Delta = 0, \quad UV + VU = 0.$$

The equations (14) and (15) can be represented in the equivalent form

$$\begin{aligned} \bar{\Delta} \exp \left\{ \frac{i}{\hbar} S \right\} &= 0, \\ \tilde{\Delta} \exp \left\{ \frac{i}{\hbar} X \right\} &= 0 \end{aligned} \quad (23)$$

with the help of the operators

$$\bar{\Delta} = \Delta + \frac{i}{\hbar} V, \quad \tilde{\Delta} = \Delta - \frac{i}{\hbar} U,$$

having the algebraic properties

$$\bar{\Delta}^2 = 0, \quad \tilde{\Delta}^2 = 0, \quad \bar{\Delta} \tilde{\Delta} + \tilde{\Delta} \bar{\Delta} = 0.$$

By the nilpotency (22) of the operator U (19), any functional

$$X = U \Psi[\Phi] \quad (24)$$

with a fermionic functional $\Psi[\Phi]$ is a solution of the equation (15). The above expression (24) has the exact form of the gauge-fixing functional applied by the method of superfield BRST quantization [9].

By virtue of eqs. (14) and (15), the integrand in eq. (13) is invariant under global supersymmetry transformations with an anticommuting parameter μ ,

$$\begin{aligned}\delta\Phi^A(\theta) &= \mu U\Phi^A(\theta) + (\Phi^A(\theta), X - S)\mu, \\ \delta\Phi_A^*(\theta) &= \mu V\Phi_A^*(\theta) + (\Phi_A^*(\theta), X - S)\mu.\end{aligned}\tag{25}$$

Eqs. (25) are the transformations of BRST symmetry in the framework of the modified superfield BRST quantization.

By virtue of eqs. (14), (15), (22) and (25), the vacuum functional $Z \equiv Z_X$ in eq. (13) does not depend on the choice of the gauge boson, $Z_{X+\delta X} = Z_X$, which, consequently, ensures [11] the gauge-independence of the S -matrix.

The method of modified superfield BRST quantization permits to generalize the BV quantization scheme with respect to the gauge-fixing procedure. In fact, consider the component representation of the superfields $\Phi^A(\theta)$ and super-antifields $\Phi_A^*(\theta)$

$$\Phi^A(\theta) = \phi^A + \lambda^A\theta, \quad \Phi_A^*(\theta) = \phi_A^* - \theta J_A.$$

The set of the variables ϕ^A , ϕ_A^* , λ^A and J_A coincides with the complete set of variables applied by the BV method, where the components J_A , $\varepsilon(J_A) = \varepsilon_A$, are identified with the sources to the fields ϕ^A .

In the component form, the antibracket (16) and the operator Δ (17) coincide with the corresponding objects (1) and (2) of the BV method.

Let us now represent the integration measure (20) in the component form

$$d\Phi d\Phi^* \rho[\Phi^*] = d\phi d\phi^* d\lambda dJ \delta(J).$$

Solutions of the generation equation that determine the action S when $J_A = 0$ may be sought among solutions of the master equation (3) applied by the BV method, since the operator V (18)

$$V = -J_A \frac{\delta}{\delta\phi_A^*}$$

vanishes when $J_A = 0$.

Let us choose the functional S to be independent of λ^A . Taking into account the component form of $\Phi^*\Phi$ in eq. (21)

$$\Phi^*\Phi = \phi_A^*\lambda^A - J_A\phi^A,$$

we obtain the following representation of the vacuum functional (13):

$$Z = \int d\phi d\phi^* d\lambda \exp \left\{ \frac{i}{\hbar} \left[S(\phi, \phi^*) + X(\phi, \phi^*, \lambda) + \phi_A^*\lambda^A \right] \right\}. \tag{26}$$

The above result (26) may be considered as a natural extension of the BV quantization procedure to a more general case of gauge-fixing. In fact, the

functional $X = U\Psi[\Phi]$ is a solution of the generating equation (15). Note that the component representation of the operator U (19) reads

$$U = -(-1)^{\varepsilon_A} \lambda^A \frac{\delta_l}{\delta \phi^A}.$$

Let us choose the functional Ψ to be independent of the fields λ^A , $\Psi = \Psi(\phi)$. Then we find that the gauge-fixing functional X

$$X(\phi, \lambda) = -\frac{\delta \Psi(\phi)}{\delta \phi^A} \lambda^A \quad (27)$$

becomes identical with the gauge applied by the BV quantization method, and therefore in the above particular case (27) of solutions that determine X the vacuum functional (26) of the modified superfields BRST formalism coincides with that of the BV method (11). This means, given the established gauge-independence within the BV approach as well as within the modified superfield BRST formalism, the coincidence of the S -matrices in both these methods, including the case when the gauge-fixing of the BV method is extended to an arbitrary (admissible) gauge fermion Ψ in the symmetry transformation (7).

Let us show, however, that the extension of the gauge-fixing procedure of the BV method obtained as a generalization of the gauge fermion does not coincide with that provided by the modified superfield BRST formalism. Consider the generalization of the BV vacuum functional (11) following from eq. (5) with the symmetry transformation (7) extended to the case of the gauge fermion chosen as a functional of fields and antifields, $\Psi = \Psi(\phi, \phi^*)$. Then the original vacuum functional (5) can be represented in the form

$$Z = \int d\phi d\phi^* d\lambda \exp \left\{ \frac{i}{\hbar} \left[S(\phi, \phi^*) + S_\Psi(\phi, \lambda) + \phi_A^* \lambda^A \right] \right\}, \quad (28)$$

where the corresponding gauge-fixed part $S_\Psi(\phi, \lambda)$ of the quantum action,

$$\begin{aligned} \exp \left\{ \frac{i}{\hbar} S_\Psi(\phi, \lambda) \right\} &= \int d\phi' d\phi'^* \delta \left(\phi - \phi' + \frac{\delta \Psi}{\delta \phi'^*} \right) \delta(\phi'^*) \times \\ &\times \exp \left\{ \frac{i}{\hbar} \left[- \left(\phi'^* + \frac{\delta \Psi}{\delta \phi'} \right) \lambda + i\hbar \Delta' \Psi \right] \right\}, \end{aligned}$$

does not depend on the antifields. In this sense, the introduction of gauge with the help of the bosonic functional X (15) and (26) in the framework of the modified superfield BRST approach can be considered as a formal extension of the gauge-fixing procedure introduced by means of the symmetry transformation (7) within the BV quantization method.

Note that gauge-fixing in terms of the bosonic functional X (15), or (23), also leads to a generalization of the usual Ward identities. Indeed, define, as

we return to the vacuum functional (26), the extended generating functional $Z(J, \phi^*)$ of Green's functions by the rule

$$Z(J, \phi^*) = \int d\phi' d\phi^{*'} d\lambda \exp \left\{ \frac{i}{\hbar} \left[S(\phi', \phi^{*'}) + X(\phi', \phi^{*'}, \lambda) + (\phi^{*'} - \phi^*)\lambda + J\phi' \right] \right\}, \quad (29)$$

where we have introduced the sources J_A to the fields ϕ^A .

In the particular case when the bosonic functional X , by analogy with the BV method (28), does not contain dependence on the antifields, $X = X(\phi, \lambda)$, the generating equation (23) that determines X takes on the form

$$U \exp \left\{ \frac{i}{\hbar} X \right\} = 0. \quad (30)$$

From eqs. (29) and (30), with allowance for eq. (3), follow the well-known Ward identities for gauge theories

$$J_A \frac{\delta}{\delta \phi_A^*} Z(J, \phi^*) = 0.$$

When extending eq. (30) to the case (23) that corresponds to the presence of antifields in the gauge-fixing functional, $X = X(\phi, \phi^*, \lambda)$, the form of the corresponding Ward identities is modified to

$$J_A \frac{\delta}{\delta \phi_A^*} Z = \frac{1}{i\hbar} J_A \left\langle \frac{\delta X}{\delta \phi_A^{*'}} \right\rangle Z,$$

where we have used the notation $\left\langle \delta X / \delta \phi_A^{*'} \right\rangle$ for the vacuum expectation value of the functional $\delta X / \delta \phi_A^{*'}$, $X = X(\phi', \phi^{*'}, \lambda)$.

4 Conclusion

In this report, we have considered the modified scheme [10] of superfield Lagrangian quantization based on the principles of BRST symmetry and the gauge-independence of the S -matrix. We have shown that the formalism [10] provides a generalization of the gauge-fixing procedure of the BV quantization method [6], different from the natural extension of the gauge-fixing inherent in [6]. Thus we conclude that the above two extensions of the BV approach provide off-shell different realizations of the principles of BRST symmetry and gauge-independence, while leading to identical S -matrices.

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